1993 Q6 Nett Force = mass × accel

force $= m\omega^2 x$ if performing SHM about $x=0$

$\Rightarrow \frac{7m}{16} = m\omega^2 A$

Std $A=4$ $\Rightarrow \frac{7m}{16} = m\omega^2 4$

$\Rightarrow \frac{7}{64} = \omega^2 \Rightarrow \omega = \frac{\sqrt{7}}{8}$. OR 0·33 rad/sec

∴ (i) Period $= \frac{2\pi}{\omega} = \frac{2\pi}{\frac{\sqrt{7}}{8}} = \frac{16\pi}{\sqrt{7}}$, OR 19 seconds.

(ii) Either: $x=0$ | | $t = \frac{\pi}{\sqrt{7}}$

$x = A\sin\omega t$

$x = 4\sin\left(\frac{\sqrt{7}}{8}\frac{2\pi}{\sqrt{7}}\right)$

$x = 4\sin\left(\frac{\pi}{4}\right)$

$x = 4\left(\frac{1}{\sqrt{2}}\right) = 2\sqrt{2}$

∴ $v^2 = \omega^2(A^2 - x^2)$

$v^2 = \left(\frac{\sqrt{7}}{8}\right)^2\left(4^2 - (2\sqrt{2})^2\right)$

$v^2 = \left(\frac{\sqrt{7}}{8}\right)^2(16-8)$

$v^2 = \left(\frac{\sqrt{7}}{8}\right)^2(8)$

$v^2 = \frac{7}{8} \Rightarrow v = \sqrt{\frac{7}{8}} = .94$ secs

OR

$x = A\sin\omega t$

$\frac{d}{dt} \Rightarrow \frac{dx}{dt} = A\omega\cos\omega t$.

$v = A\omega\cos\omega t$.

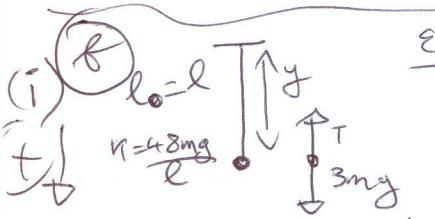
$v = 4\left(\frac{\sqrt{7}}{8}\right)\cos\left(\frac{\sqrt{7}}{8}\frac{2\pi}{\sqrt{7}}\right)$

$= \frac{\sqrt{7}}{2}\cos\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{7}}{2}\frac{1}{\sqrt{2}}$

$= \frac{\sqrt{7}}{2\sqrt{2}}$

$= \frac{\sqrt{7}}{\sqrt{8}}$ secs.

(i) 

$T = \kappa(y-l)$

Equil position: $\Sigma F = 0$.

$-T + 3mg = 0$

$-\kappa(y-l) + 3mg = 0$.

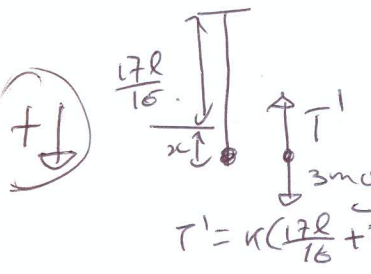
$-\frac{48mg}{l}(y-l) = -3mg$

$-y + l = \frac{-l}{16}$

$\Rightarrow y = \frac{17l}{16}$ at equil position.

+ ↓ cos x increases in this direction

Typical position:



$\Sigma F = ma$.

$-T + 3mg = 0$.

$T' = \kappa\left(\frac{17l}{16} + x - l\right) = \kappa\left(\frac{l}{16} + x\right)$

$= \frac{48mg}{l}\left(\frac{l}{16} + x\right)$

$-\frac{48mg}{l}\left(\frac{l}{16} + x\right) + 3mg = 3ma$

$-3mg - \frac{48mg}{l}x + 3mg = 3ma$.

$-\frac{48g}{l}x = 3a$

$-\frac{16g}{l}x = a$

SHM about $x=0$ with $\omega = \sqrt{\frac{16g}{l}} = 4\sqrt{\frac{g}{l}}$

$\Rightarrow$ Period $T = \frac{2\pi}{\omega} = \frac{2\pi}{4\sqrt{\frac{g}{l}}} = \frac{\pi}{2}\sqrt{\frac{l}{g}}$ secs. $\Rightarrow T = \frac{\pi}{2}\sqrt{\frac{l}{g}}$ secs.